

### Chapter 13: Analysis of Social Balance

This chapter will review an experiment on social balance theory. You will learn how to analyze a simple algebraic model in Excel, and how to fit such a model to empirical data. The results will show excellent agreement between the model and data obtained in a Web experiment.

The multiplying model predicts a crossover interaction. This is an excellent example of a model in which there can be no main effects and yet there is a very strong interaction. This chapter will illustrate this type of interaction with the social psychology of Heider's balance theory (Heider, 1946).

You have probably heard the expressions, “Any friend of my friend is a friend of mine,” or “my enemy’s enemy is my friend.” These statements are expressions of social balance (Heider, 1946). A social structure would be balanced if there were two cliques, in which everyone within each clique likes everybody else in the clique and dislikes everyone in the rival clique (Cartwright & Harary, 1956).

When there is not balance, the results are tragic, in both literature and life. For example, in *Romeo and Juliet*, the Capulets all loved each other and hated the Montagues, and the Montagues all loved each other and hated all the Capulets. But then, Romeo (Montague) and Juliet (Capulet) fell in love, and you know the story—it ends in grief. A real tragedy that occurs all too often is when children are caught in a bitter divorce. The child loves both mom and dad, but they now hate each other—to restore balance, the child must take one side or the other, because being caught in the middle causes distress and dismay.

Load the file, *heider.htm*, which was created with factorWiz, and complete the experiment, if you have not done so already. In this experiment, there are three people, You, Bill, and John. The independent variables are how much you like Bill and how much Bill likes John. The dependent variable is the judgment of how much you think you will like John.

The diagram in Figure 13.1 can be used to represent the social situation of this experiment. Insert Figure 13.1 about here.

You might love or hate Bill, and Bill might love or hate John, and the question is, how much will you like John? According to Balance theory, if you love Bill, then if Bill loves John, you will love John. Those who endeavor to sell basketball shoes say, “Michael Jordan likes these shoes.” So, if you like Mike, you should like these shoes.

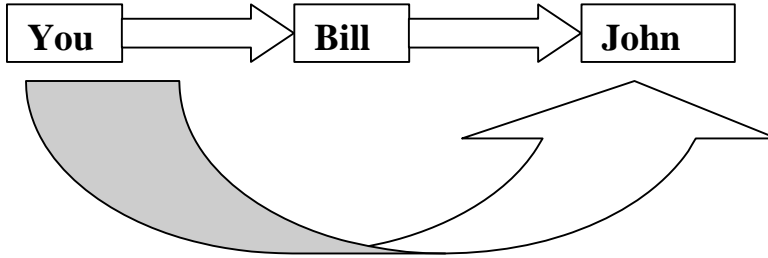
If you love Bill, and if Bill hates John, then balance occurs if you hate John. However, if you hate Bill, and Bill loves John, you also suspect you will dislike John. (Any friend of your enemy is probably your enemy. When people try to convince somebody that something is bad, they say, “Oh, yes, that's what Hitler liked.”)

Let positive numbers represent liking and negative numbers represent disliking. The following multiplication represents a balanced situation:

$$\text{Your liking of John} = (\text{Your liking of Bill}) \times (\text{Bill's Liking of John}) \quad (13.1)$$

In a state of balance, the product of the signs, going around the diagram in Figure 13.1 would be positive. Therefore, the sign of your liking of John should be the product of the other two.

Figure 13.1. The experiment asked you to judge how much you expect to like John based on your liking for Bill, and Bill's liking for John. According to balance theory, the diagram will be balanced if the product of the three signs is positive. This chapter will investigate a stronger hypothesis; namely, that your liking of John is (the product of) your liking for Bill times Bill's liking of John.



Think of how the multiplication works. The product of two positives is positive (a friend of a friend is a friend of mine), and the product of two negatives is positive (my enemy's enemy is my friend). However, the product of positive and negative is negative (my friend's enemy is my enemy) and the product of a negative and a positive is negative (my enemy's friend is my enemy).

### A. Graph of Multiplication

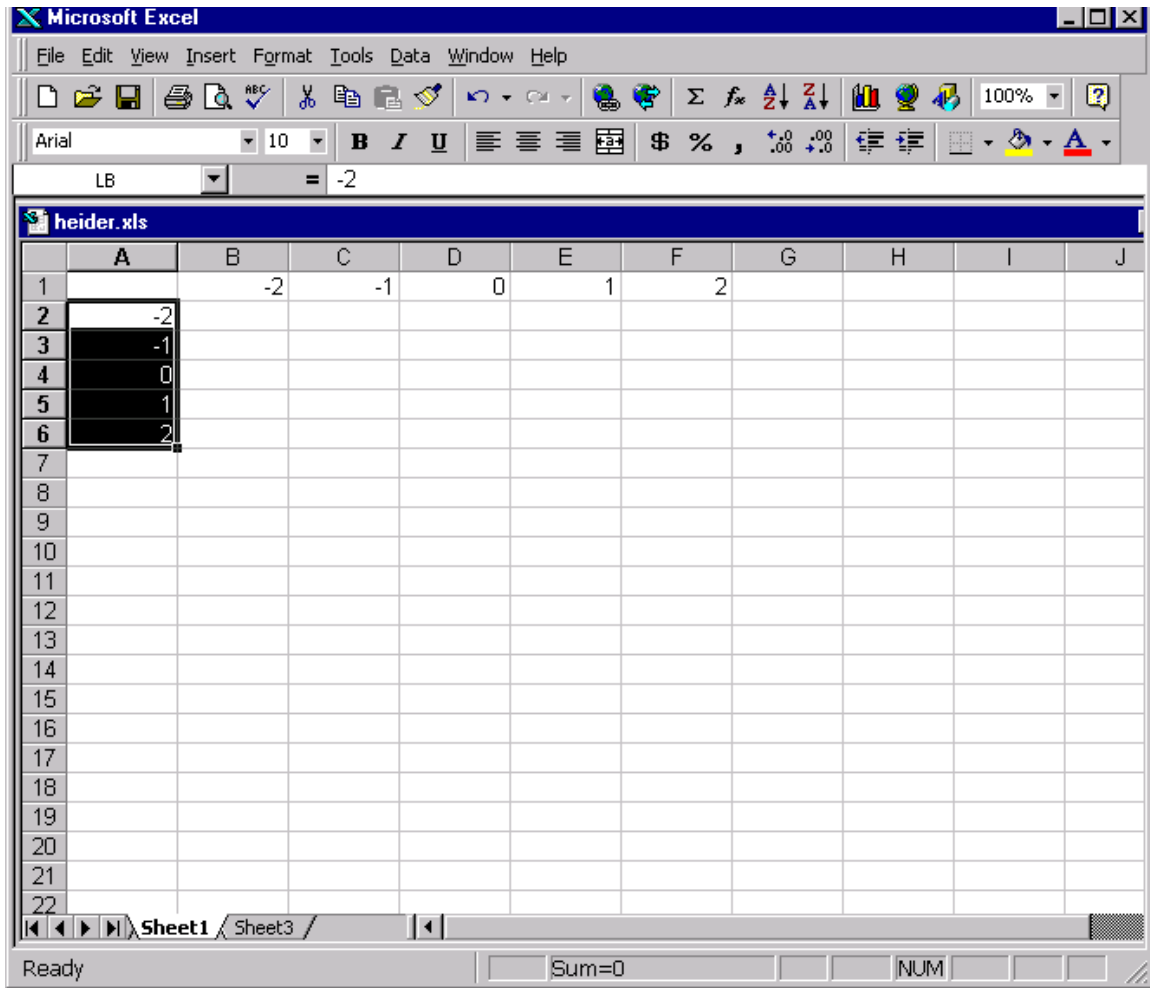
You can follow the same procedures used in Chapter 12 to test if there is an interaction between two factors. The dependent variable in this experiment is the judgment of liking. The independent variables are (Row) your liking of Bill, and (Column), Bill's liking of John.

The theory is multiplicative, so it predicts a special kind of interaction between two factors. This interaction is sometimes called a Linear by Linear, or bilinear interaction. Imagine a graph of the function,  $y = ax$ . When  $a = 1$ ,  $y = ax$  plots a straight line that goes through the origin, with a slope of 1. When  $a = 2$ , the graph of  $y = ax$  is a straight line whose slope is 2. The two lines intersect at the origin [i.e., at (0, 0)]. Now add a line with  $a = -1$ ; this line decreases, but all three lines still intersect at the origin.

To get an idea what the graph of  $y = ax$  looks like, you can draw a graph in Excel. Start on a fresh workbook, and enter the numbers  $-2, -1, 0, 1, 2$  in cells A2 to A6 and also in cells B1 to F1. Next, select the cells A2 to A6. Name these cells LB (for your liking of Bill). To name the selected cells, type in LB in the *Name Box* (directly above Cell A1 on the formula line). Then hit return. If you had cells A2 to A6 selected, then you have just renamed the collection as LB, as illustrated in Figure 13.2.

Insert Figure 13.2 about here.

Figure 13.2. Name the row factor LB, by selecting A2:A6 and typing LB in the *Name Box* (Shown here above the filename, above Cell A1). Hit *Enter*.



Next, select B1 to B6, and name them BLJ (for how much Bill likes John). Next, click in cell B2, and type “=LB\*BLJ”, (without the quotes), as shown in Figure 13.3.

Insert Figure 13.3 about here.

When you hit the *Enter* key, the number 4 will appear in the cell. What has happened is that you have multiplied  $-2$  times  $-2$  and gotten 4. However, you used a formula so now you can use *AutoFill* to fill in the entire array. After hitting return in cell B2, put the cursor in that cell (B2) again, and move it to the lower right until the *AutoFill* handle (+) appears, then drag down to fill the column; next (with the column selected), drag the whole column to the right to fill the entire array. The array will match Figure 13.4.

Insert Figure 13.4 about here.

Notice that each entry in Figure 13.4 is the product of LB and BLJ. The next task is to graph it. Select the whole matrix (i.e., select A1:F6), then click the chart icon on the toolbar (or select *Chart* from the **Insert** Menu). Then use the Wizard to make the figure following the procedures described in Chapter 12, and insert it as a new worksheet. After formatting lines and styles, the figure appears in Figure 13.5.

Insert Figure 13.5 about here.

Figure 13.5 shows what one can expect to see, if the two independent variables combine by multiplication. The graph shows  $y = ax$  with  $x = -2$  to 2 and  $a = -2$  to 2. This fan of crossing lines is sometimes called the “spider” of multiplication. However, notice that within the domain of positive numbers (the upper right quadrant of Figure 13.5), multiplication appears as a diverging fan of curves, and in the domain of negative values of  $x$  (lower left quadrant), it produces converging curves.



Figure 13.4. Click in cell B2, move the cursor to the lower right corner until the + appears. Drag down, using *AutoFill* to complete the column. With the column still selected, drag to the right, filling in the entire matrix. Each entry is the product of the entries in the first row and the first column, i.e., the product of  $LB \cdot BLJ$ .

Microsoft Excel

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heider.xls

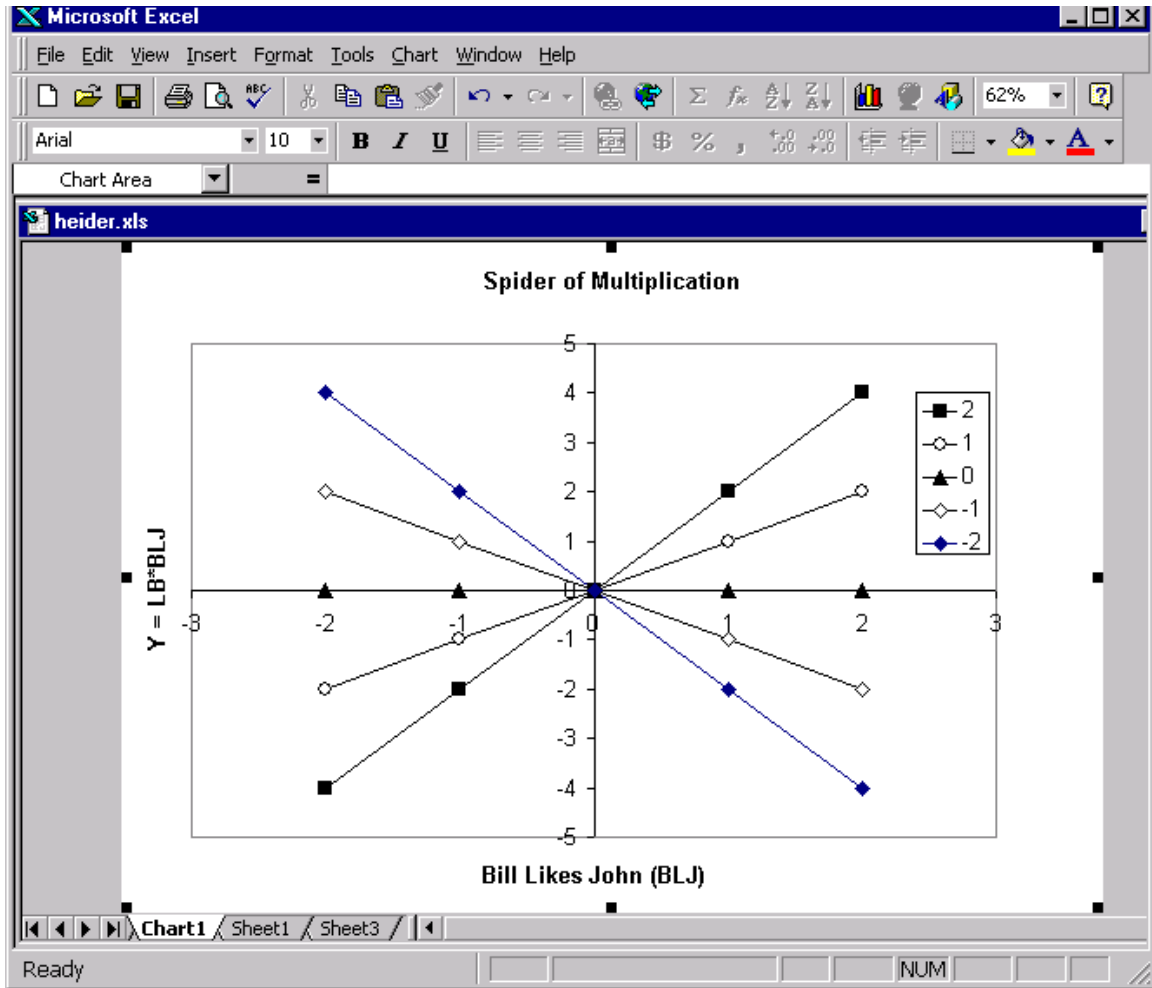
	A	B	C	D	E	F	G	H	I	J
1		-2	-1	0	1	2				
2	-2	4	2	0	-2	-4				
3	-1	2	1	0	-1	-2				
4	0	0	0	0	0	0				
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Sheet1 Sheet3 /

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Figure 13.5. Intersecting lines representing multiplication form a “spider” pattern.



Double click on the tab of *Sheet1* and rename this sheet *theory*. Now that you understand what the theory predicts, you will examine the data, to see if they resemble these predictions. The next two sections allow you to practice techniques that were explained more fully in Chapter 12, to analyze the data from this experiment.

### **B. Filter the Data and Find Column Means**

Follow the same procedures here as in Chapter 12 to filter and clean the data. First, open *clean.xls* or *clean.csv*, turn on *AutoFilters*, and choose “heider” from the list of experiments in the first column. Second, select, *Copy* and *Paste* the data to a new sheet in the workbook containing the predictions of multiplication. Add column labels. Third, use conditional formatting for the judgments to find values outside of the range from 1 to 9. Fourth, find the column mean in the first column of data, and then use *AutoFill* to get means for all of the 25 columns of data (see Chapter 12). Save this workbook as *heider.xls*. A copy of the completed file is on the CD for you to compare with your work; a cleaned version of the data, corrected for out of range responses and other violations of instructions, is on a separate sheet in this file labeled *clean data*. The file will now appear as in Figure 13.6. Insert Figure 13.6 about here.

### **C. Copy the Means to Make the Matrix and Graph**

Next, copy the means to a new worksheet, and *Paste Special* into B2, be sure to use *Paste Link*; that way, any change to the data will be reflected in this matrix. Then cut and paste the data in segments until the 5 by 5 matrix is completed. Next, add labels for the rows and columns. Procedures to do these tasks are explained in greater detail in Chapter 12, Section C. The result is shown in Figure 13.7.

Insert Figure 13.7 about here.



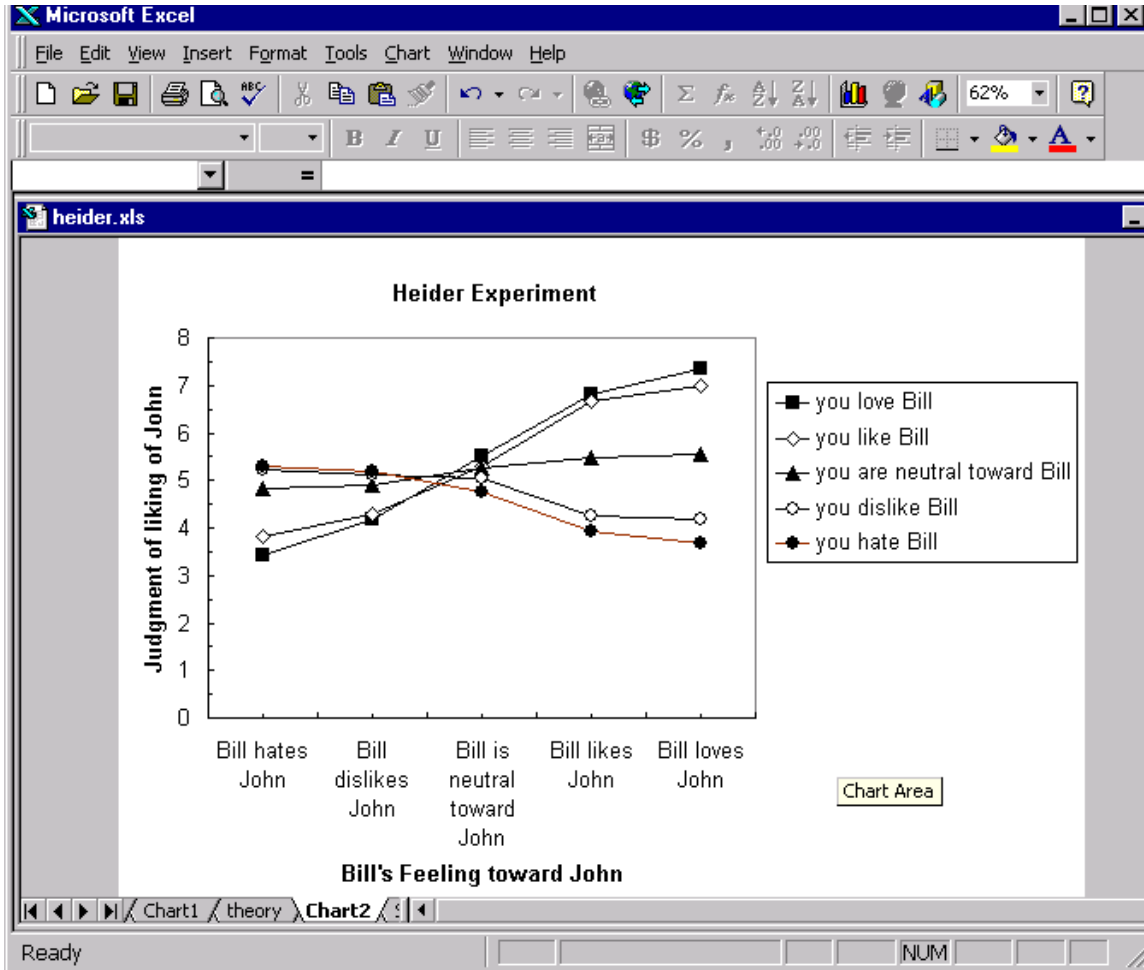


The next step is to draw a graph of the data. To draw the graph, select the matrix of means (Fig. 13.7), including the labels (A1:F6), and click the chart icon on the toolbar (or select *Chart* from the **Insert** Menu). Then follow the chart wizard (as in Chapter 12). Insert the chart on a separate sheet. To alter a chart element, recall that you can double click on it to bring up a menu, or right-click on it on a PC to bring up a menu (On the Mac, Ctrl-click does the same). The graph is shown in Figure 13.8.

Insert Figure 13.8 about here.

Note that we have a set of lines that intersect at a point near “neutral” on the liking scale. There is an obvious similarity between Figure 13.8 (showing data) and Figure 13.5 (showing predictions of the model). The next section will fit the model to the data, to solve for subjective values.

Figure 13.8. Mean judgments of your liking of John, plotted as a function of Bill’s liking of John, with a separate curve for each level of how much you like Bill. The mean judgments show a similar interaction to that shown in Figure 13.5.



#### D. How to Fit a Multiplicative Model

Notice that the lines in Figure 13.8 are not exactly linear. Also, the negative slope when *you hate Bill* is not simply the negative of the (positive) slope when *you love Bill*. In addition, being *neutral* produces a slightly positive slope, so being *neutral* is slightly positive. By fitting a model, you will estimate numerical values from the data that represent the subjective values of such terms as *like*, *love*, *neutral*, and *hate*. The model is as follows:

$$\text{Liking of John} = (\text{liking of Bill}) * (\text{Bill's liking of John}) + b$$

Notice that this model requires estimates for the rows (*liking of Bill*) the columns (*Bill's liking of John*), and the additive constant, *b*. From the graph, we see that *b* (the projection of the point where the curves cross onto the ordinate) is approximately 5, the point that was labeled *neutral* on the response scale.

You will now modify the calculations at the beginning of the chapter and use them to fit the multiplicative model to the data. Click on the tab for those calculations, named *theory*. In cell A1 of the theory matrix, type the number 5, then click in the *Name Box*, and name it B; hit return. Then click in Cell B2, and enter =LB\*BLJ +B. Hit return, and Cell B2 should be 9. Then use *AutoFill* to complete the matrix, by dragging the first column, and then dragging the column to fill the matrix. The matrix should now appear as in Figure 13.9.

Insert Figure 13.9 about here.





The next step is to calculate the sum of squared differences between the mean data and the theory matrix. To do this, copy the matrix of mean judgments (B2:F6) and use *Paste Special*, to paste the data matrix in Cells B8:F12 of the sheet for *theory*. From the *Paste Special* dialog, click *paste all*, and click *Paste Link*. The screen is shown in Figure 13.10.

Insert Figure 13.10 about here.

The next task is to put the sum of squared differences between the arrays in Cell G13. To do that, click in cell G13, and click the *Function* icon on the toolbar (or select *Function* from the **Insert** Menu). Choose *SUMXMY2* from the list of functions in the dialog box shown in Figure 13.11.

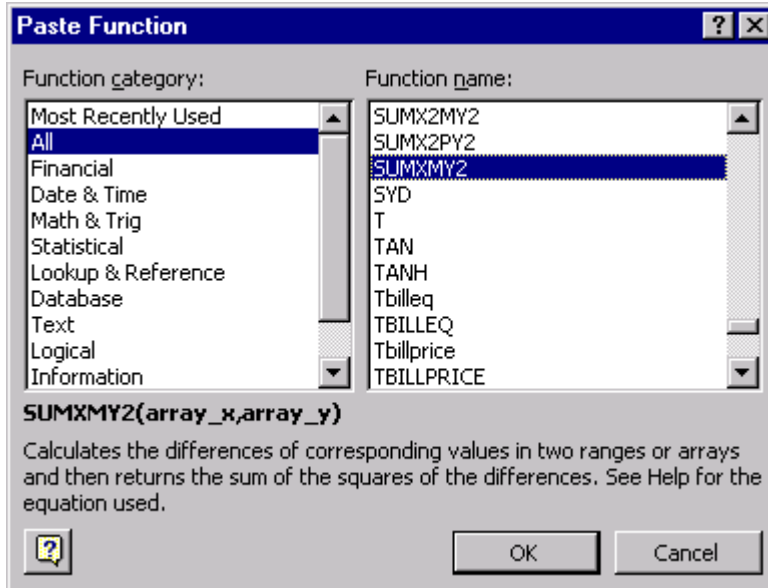
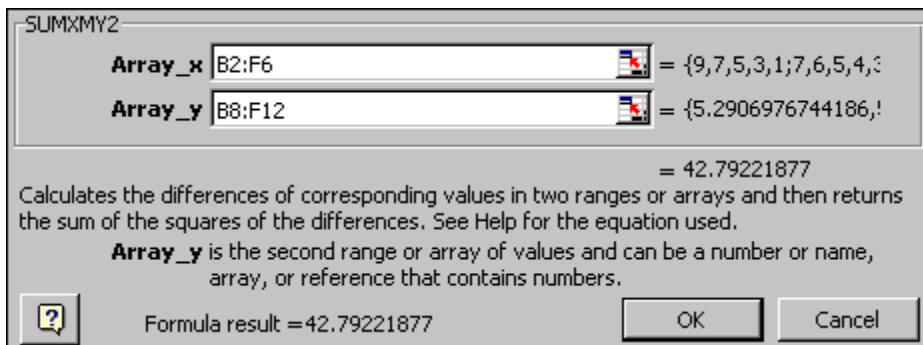
Insert Figure 13.11 about here.

Then select the cells B2:F6 (either by typing or by using the mouse to select the cells) for *Array\_x*, then click in the box for *Array\_y* and select (or type) the cells B8:F12. The dialog appears in Figure 13.12.

Insert Figure 13.12 about here.

Notice that the function dialog box describes what the function will do, and it also gives the numerical value. (If you have further questions about a function, you can click the question mark in the left lower corner of this box). When everything looks right, click *OK*. The cell G13 now contains a measure of how far the theoretical predictions are from the data means.



Figure 13.11. Paste Function dialog box. Choose *SUMXMY2*.Figure 13.12. *SUMXMY2* dialog requires specification of *Array\_x* and *Array\_y*, which can be done by typing in the cell ranges, or by selecting the cells with the mouse.

Now click on G13 (the cell containing the sum of squared differences) and select *Solver* from the **Tools** menu. The *Solver* dialog box appears (Figure 13.13). Note that the *Set Target Cell:* is Cell \$G\$13. If it is not, then type G13 in the box. Next click the button for *Min*, which will minimize the value in Cell G13. Now click in the box labeled “*By Changing Cells:*” and select Cells \$A1:\$F1, and while holding down the *Control* key, select \$A2:\$A6. These cells hold the parameters for the Row and Column and the additive constant. You could also type in this box the names of these parameters: LB, BLJ, and B, separated by commas.

Insert Figure 13.13 about here.

When everything looks right in this dialog (Figure 13.13), click the *Solve* button in the upper right of the dialog box. Pushing the *Solve* button brings up the dialog in Figure 13.14. You can explore these options later. For now, click *OK*. The window of the *theory* sheet should now appear as in Figure 13.15.

Insert Figures 13.14 and 13.15 about here.

Solver caused four things to happen on this sheet: First, the value of G13, which measures the sum of squared differences between the entries in the theory matrix and the data matrix has decreased from 42.7 to 0.35. In other words, the fit has improved. Second, the numbers  $-2, -1, 0, 1, 2$ , which were set as initial Row values (representing how much *you like Bill*) have changed to  $-0.74, -0.46, 0.37, 1.36, 1.60$  for *hate, dislike, neutral, like, and love*, respectively. Notice that if you are *neutral* toward Bill, the value is slightly positive (.37). Third, the Column values, which were originally  $-2, -1, 0, 1, 2$ , have changed to  $-0.82, -0.44, 0.31, 1.26, 1.54$  for the levels of *Bill's liking of John: hates, dislikes, neutral, likes, and loves*, respectively. Again, if Bill is neutral toward John, it is slightly positive. Fourth, the value of B has changed from 5 to 4.92.

Figure 13.13. The Solver Parameters dialog. Set the target cell (G13), click *Min* (for minimize), and specify the list of cells to change in order to minimize the value in G13.

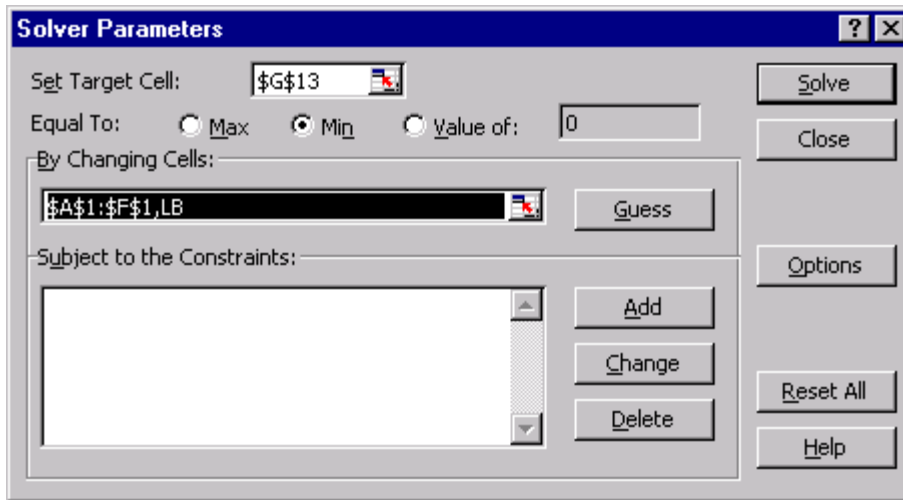


Figure 13.14. Solver results dialog box. Click *OK*.

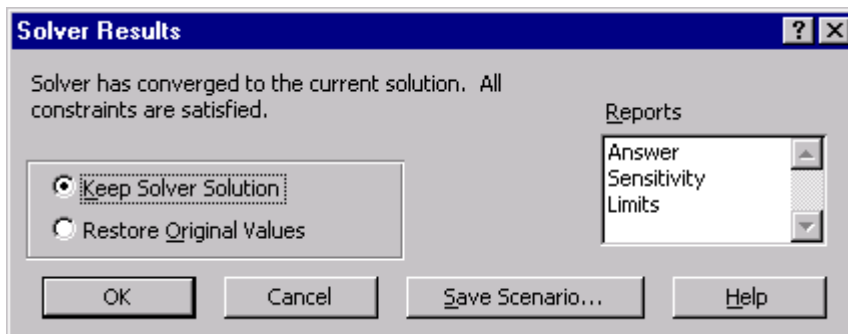
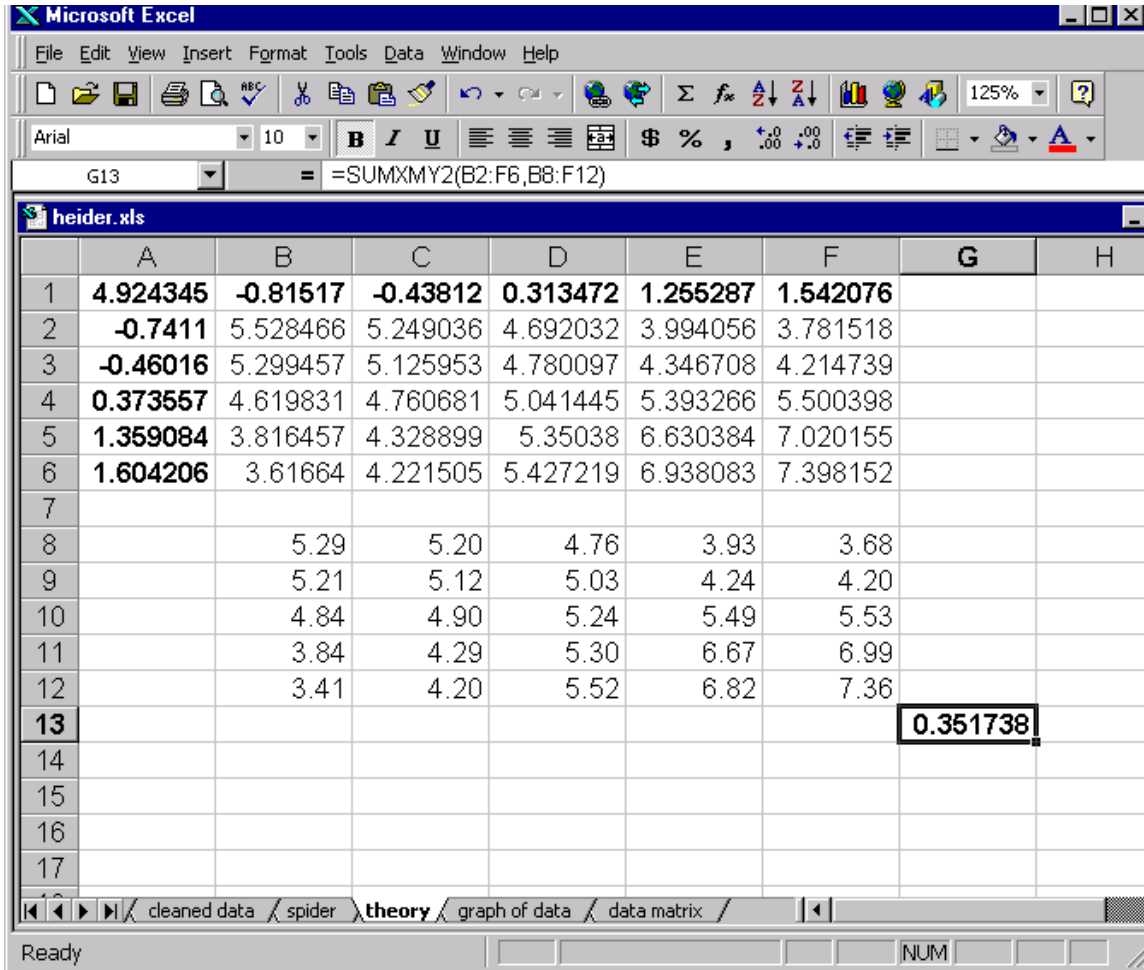


Figure 13.15. Results of Solver Solution. Compare the numbers in bold with corresponding initial values in Figure 13.10. The sum of squared differences between theory and data is only 0.35.



## E. Graphing the Predictions and the Data

To see how well a model fits the data and to allow the eye to search for systematic deviations, it is helpful to plot both the predictions and mean judgments on the same graph. To do this, select B1:F6, and while holding down the *CRTL* key, select B8:F12. (Holding down the *Control* key allows you to select several sections that may be separated on the sheet). Then click the *Chart* icon (or select *Chart* from the **Insert** menu). Select *XY scatterplot*, and click the icon *showing data points connected by lines*. When you press and hold to preview in the first step of the Wizard, the graph may appear strange, but keep the faith and continue. Click *Next*, and on the second step of the Chart Wizard, click the button that designates that data series are in Rows—the figure will now look much better.

On the same step (Step 2) of the Chart Wizard, click the tab labeled *Series*. Each row of the theory or data matrix is one series. For example, the first row of the first matrix contains the predictions for “You hate Bill.” The sixth series is the first row of actual data. Rename *Series 1* to *Series 5: You hate Bill prediction, You dislike Bill prediction, ...* to *You love Bill prediction*, respectively. Rename *Series 6* to *Series 10: You hate Bill data, You dislike Bill data, ...* to *You love Bill data*. This step is illustrated in Figure 13.16.

Insert Figure 13.16 about here.

Click *Next*. Complete the graph, which brings up Figure 13.17. As we did in Chapter 12, make the background white and eliminate the gridlines. Increase the font size of the axis labels to 16.

Insert Figure 13.17 about here.

Figure 13.16. On step 2 of the Chart Wizard, click the *Series* tab. Rename Series 1 to 10 with descriptive (informative) labels. This step requires clicking on the series label on the left, then clicking in the *Name* box and typing the name of the series. These names will be displayed on the graph.

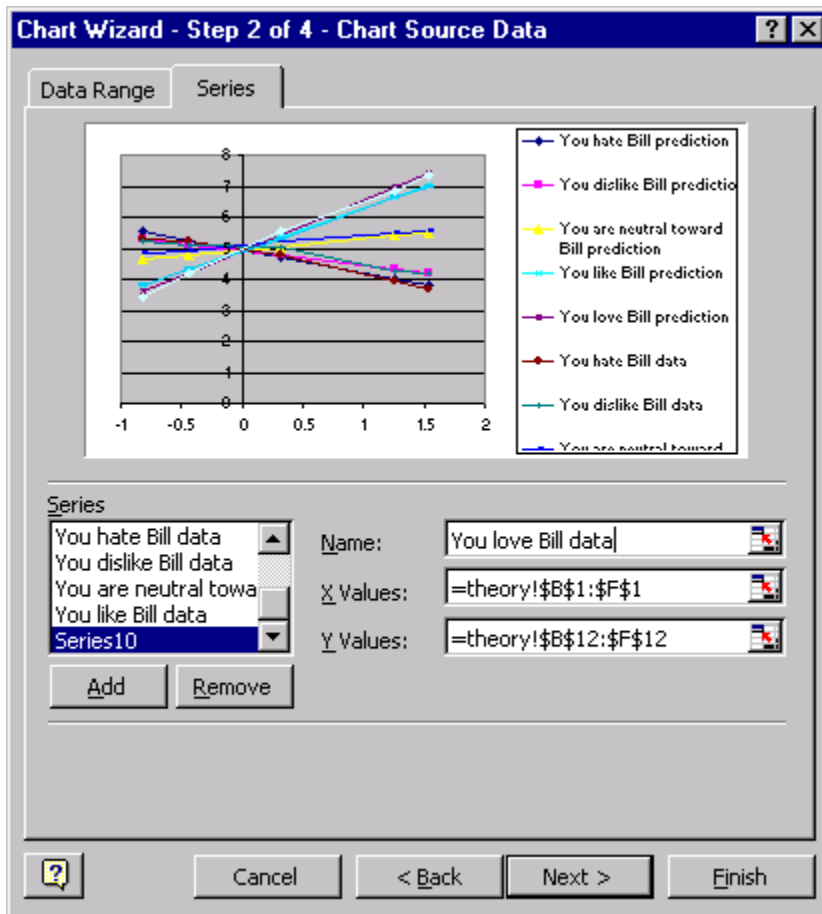
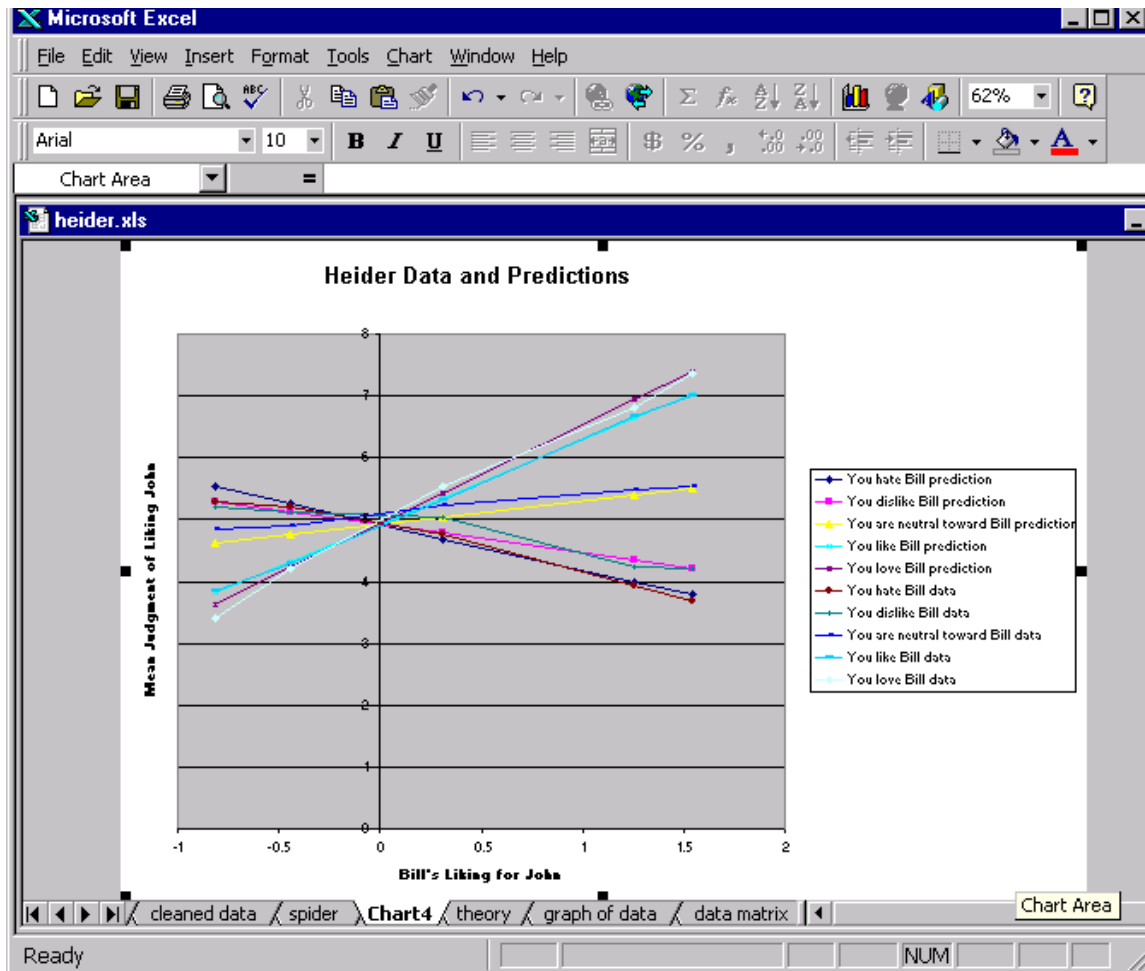




Figure 13.17. Predictions and data are plotted as a function of the estimated scale values of Bill's liking for John. There are separate curves (and markers) for data and predictions and for each level of how much you like Bill. You will remove the markers for curves of theoretical predictions and remove the lines for empirical data.



The next task is to make the prediction markers (point symbols) disappear (leaving the lines), and to make the data lines disappear (leaving the data point markers). Point the mouse arrow to the last data marker in each row. If you aim the pointer exactly at a point, a small box appears with a label for the series. When it shows a data series, double click, which brings up the dialog box shown in Figure 13.18.

Insert Figure 13.18 about here.

Notice that four things have been done in Figure 13.18. First, the *Line* has been changed to *None*. (Its color does not matter since there is no line.) Second, the *Marker* has been increased to *Size 10*. Third, the *Marker Style* has been selected as a filled circle. Fourth, the colors of both foreground and background of the Marker have been set to black. Click *OK*. Repeat this step for each series of data, using different Marker styles for each curve.

Next, select one of the series of data for predictions and double click. It brings up the same dialog box, but this time for a series of predictions. The trick now is to choose a black line, and choose “none” for Marker. Do the same for each series of predictions. The finished graph appears in Figure 13.19.

Insert Figure 13.19 about here.

Figure 13.18. Format Data Series dialog. For each series of data, choose *None* for line (left side of the *Patterns* panel) and increase Marker size (right side of the same panel) to 10. Also, make the foreground and background color black to create a filled symbol. Make the background white to make an unfilled symbol. For each series of predictions, choose *None* for *Marker*, and make the line black. Thus, each series of data will have markers without lines, and each series of predictions will have lines without markers.

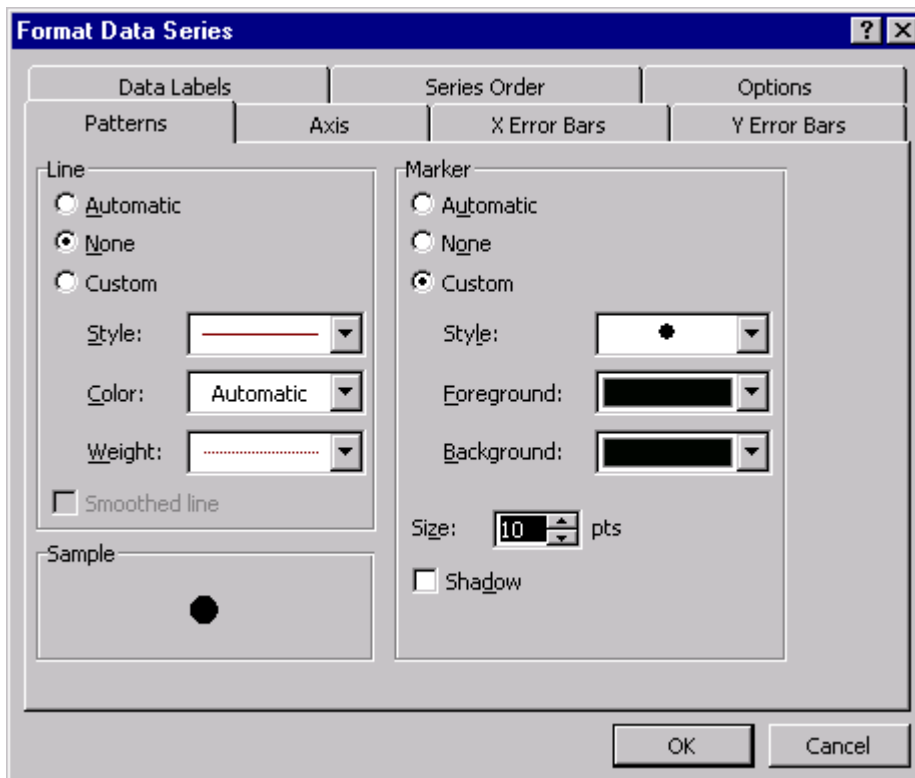


Figure 13.19. Completed figure showing mean judgments of liking for John as a function of the estimated scale values for *Bill's liking of John* with separate lines for predictions for each level of *your liking for Bill*, and markers for corresponding data. This figure also illustrates the small box that appears when the pointer aims at one of the series.

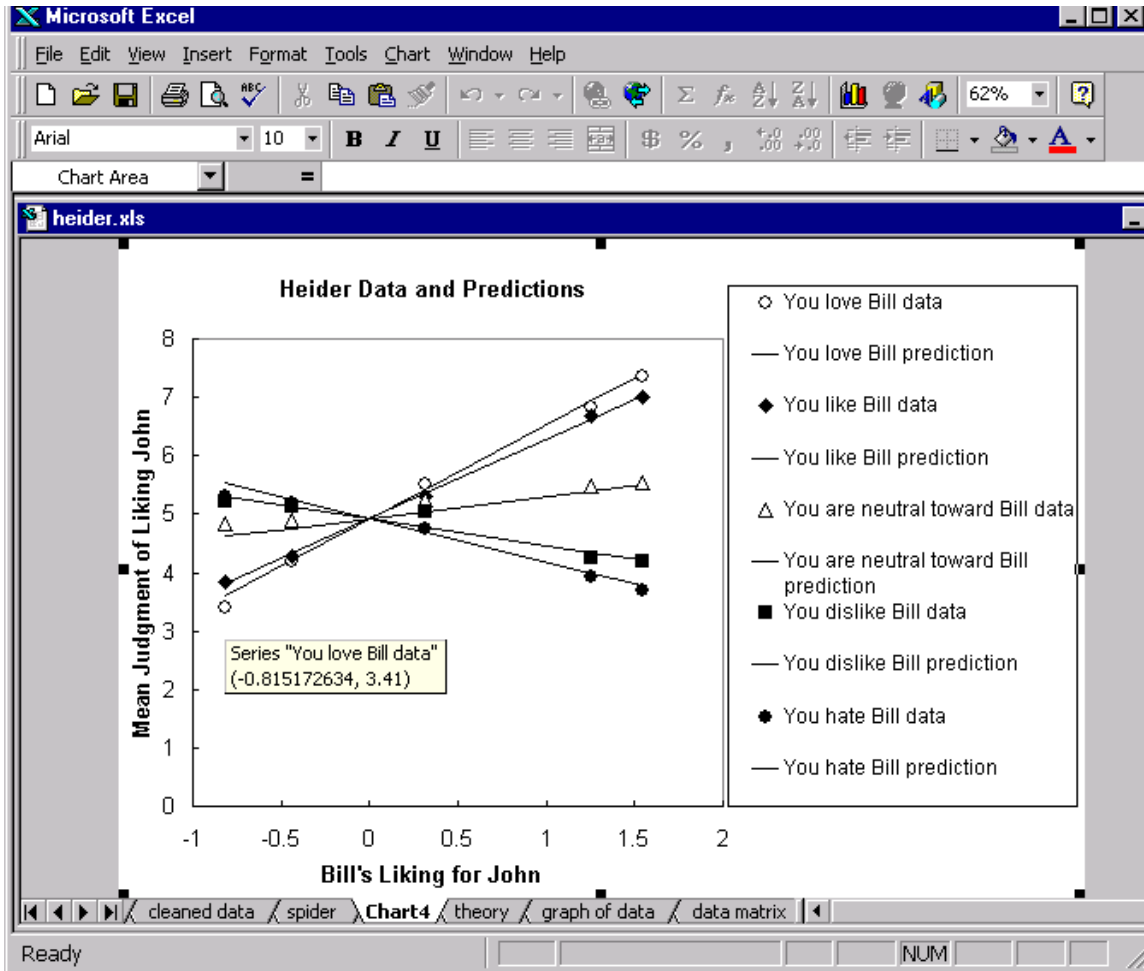


Figure 13.19 shows that the theory (lines) does a good job of fitting the data (markers). The largest discrepancies between the data and theory occur when Bill hates John. When you hate Bill and Bill hates John, the mean rating is only 5.29, so these data suggest that your enemy's enemy is not much of a friend.

Despite these discrepancies, the multiplicative representation of this social inference task appears to provide an excellent fit to the mean judgments.

The social inference task of Heider illustrates an extreme form of interaction. This interaction shows that the effect of one variable (how much Bill likes John) can be reversed by changing another variable (how much you like Bill). Although an interaction may seem a complicated idea, these data also show that a simple algebraic model (multiplication) predicts this pattern quite well. The model represents a process that people do without having to calculate on paper or count on their fingers. People don't need to make any calculations, nor are people aware that they made any calculations, because their brains make this calculation (which can be well approximated by multiplication) without interfering with speech or verbal train of thought. Thus, multiplication in this case represents how the mind works to produce judgments, but not what people report of their verbal thoughts.

## **F. Summary**

In this chapter, you learned how to graph a theory, fit it to the data, and plot both theory and data on the same graph. You analyzed data from an experiment on social balance and found that a multiplicative model provides an excellent fit to the data.

## G. Exercises

1. Subdivide the data for males and females, using Excel's *AutoFilters*. Analyze the data separately for males and females, repeating the same analyses presented in this chapter. Do the conclusions of this study depend on gender?
2. Create a theoretical matrix of ratios. Use the integers from 1 to 7 as the levels of both Row and Column. Label the rows, SR, and label columns, SC. Then compute SC/SR. Use *AutoFill* to complete the array. Plot this matrix twice: once with data in rows and once with data in columns.
3. Use SPSS to calculate an ANOVA for the Heider data (the methods are described in Chapter 12). Are main effects and interactions statistically significant?
4. How many individuals show the same crossover interaction as shown in the means? Use Excel to count (see Chapter 12 for the techniques).
5. If you understand the analyses of this chapter, then you should be able to analyze the data for adverbs and adjectives, replicating the Cliff (1959) experiment. Cliff's theory is that adverbs multiply the value of adjectives. If adverbs have all positive values, then the data should appear as a subset of the "spider" of multiplication (Figure 15.5 with positive slopes). The experimental materials are included on the CD as *AdjAdv.htm*. The experiment was created with the help of factorWiz. On the CD that accompanies this book is included a data file, *AdjAdv.csv*. These data were collected with a 4 by 5 design, where the four adverbs (*slightly*, *no adverb*, *very*, and *extremely*) were paired with 5 adjectives (*mean*, *noisy*, *blunt*, *practical*, and *understanding*). Because the adverb\*adjective combination is theorized to follow

multiplication, you should be able to apply the same techniques of this chapter to analyze these data.

6. Project idea: Think of judgment tasks that might conform to a multiplicative relationship between the independent variables. Here are some examples: Shanteau (1974) used a multiplicative model to describe how probability phrases combine with the values of prizes to determine the combined value. How much tip should you leave as a function of the size of the bill and the quality of the service? How nervous would you be to present a talk, based on the size and status of the audience? (Would you be more nervous to give a talk to a concert hall holding 200 judges or to a small class of 5 undergraduates?) How credible would an accusation against a defendant be as a function of the number and status of the witnesses who accuse him? (The witnesses might be convicted felons, factory workers, schoolteachers, or medical doctors). Design an experiment on one of these tasks (or another that you think might fit multiplication), construct the experiment with factorWiz, collect the data, and analyze them for a project.