True and Error Model of Response Variability—Part 1

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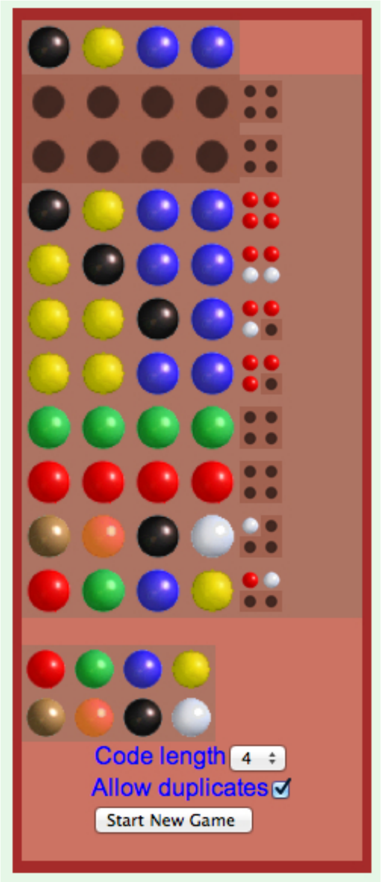
Critical Test



Outline

- Allais paradox is a critical test of EU
- Apparent violations; statistical tests.
- However, other reasonable error models allow "significant" changes that mean nothing with respect to our theory if error allowed.

Allais (1953) "Constant Consequence" Paradox

Called "paradox" because preferences contradict Expected Utility.

A:	\$1M for sure	$\succ$	*B*:	.10 to win \$5M .89 to win \$1M .01 to win \$0
C:	.11 to win \$1M .89 to win \$0	$\prec$	*D:*	.10 to win \$5M .90 to win \$0

Distracting Issues

- Very Large consequences
- Hypothetical consequences
- Sure thing (“certainty effect”)
- Three-branch gamble

Allais CC Paradox (*JMP* '04)

- Choose Between:

  $S = (\$40, 0.2; \$2, 0.8)$

  $R = (\$98, 0.1; \$2, 0.9)$

- Choose Between:

  $S' = (\$98, 0.8; \$40, 0.2)$

  $R' = (\$98, 0.9; \$2, 0.1)$

EU Model

P1: $$U(G) = \sum_{i=1}^{n} p_i u(x_i)$$

P2: $$S \succ R \Leftrightarrow U(S) > U(R)$$

(Except for “error”)

Is EU Violated?

- EU implies:
- *S* preferred to *R*
- If and only if
- *S'* preferred to *R'*

EU and deeper analysis

- The proof that EU cannot show Allais paradoxes follows from the model:
- $EU = \sum u(x)p(x)$
- However, we can decompose the paradox into three components: transitivity, coalescing, and restricted branch independence.

Coalescing

- $(x, p; x, q; y, r) \sim (x, p + q; y, r)$

- $(x, p; y, q, y, r) \sim (x, p; y, q + r)$

Restricted Branch Independence

- Weaker than Savage's "Sure Thing" Axiom, or "independence" axiom of von Neumann & Morgenstern.

- $S = (x, p; y, q; z, r) \succ R = (x', p; y', q; z, r)$

iff

- $S = (x, p; y, q; z', r) \succ R = (x', p; y', q; z', r)$

Transitivity

If A $\succ$ B and B $\succ$ C,

then A $\succ$ C.

If A $\succ$ B and B ~ C,

then A $\succ$ C.

Analysis of the Paradox:

$$(\$40, .2; \$2, .8) \succ (\$98, .1; \$2, .9)$$

$\Leftrightarrow$ (Coalescing & Trans)

$$(\$40, .1;\$40, .1;\$2, .8) \succ (\$98, .1;\$2,.1;\$2, .8)$$

$\Leftrightarrow$ (RBI)

$$(\$40, .1;\$40, .1;\$98, .8) \succ (\$98, .1;\$2,.1;\$98, .8)$$

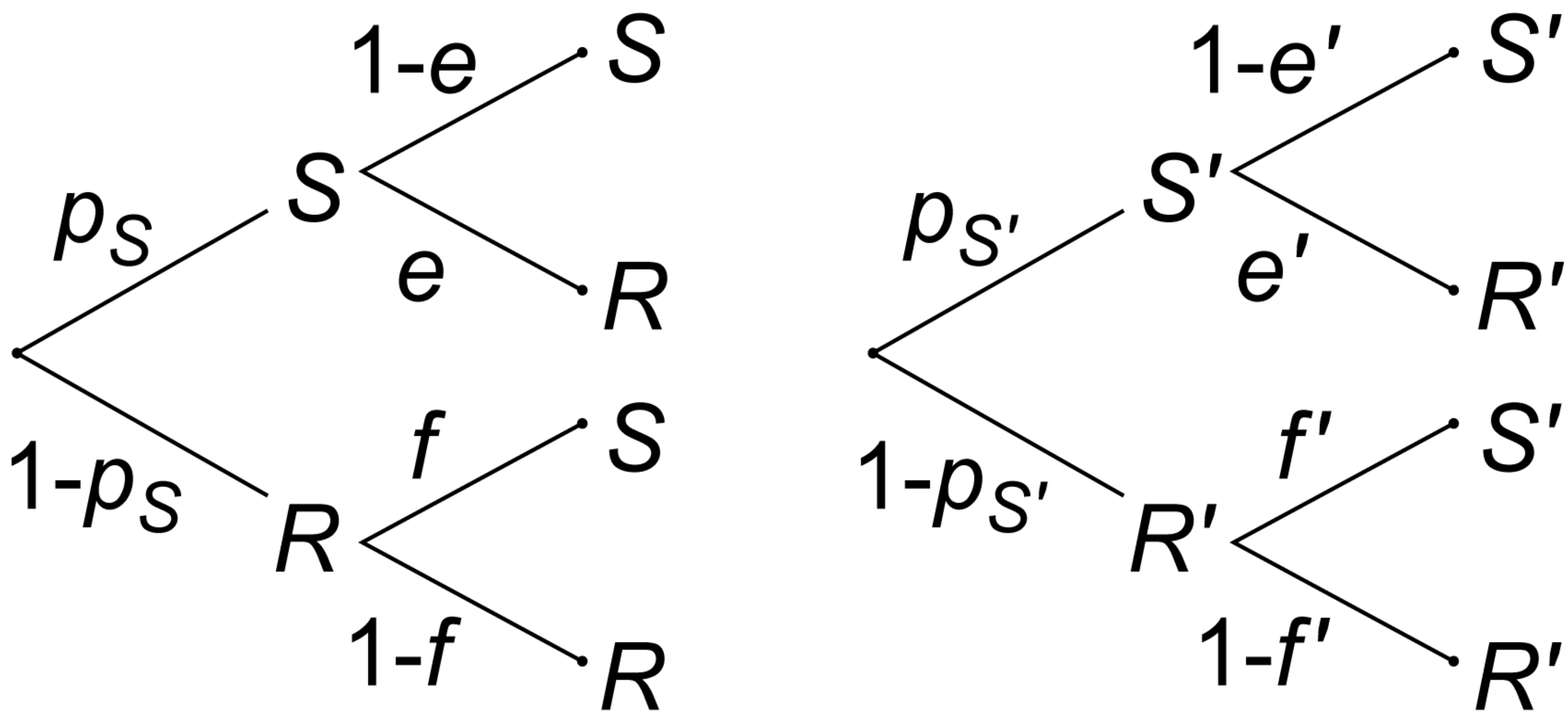
$\Leftrightarrow$ (Coalescing & Trans)

$$(\$98, .8; \$40, 0.2) \succ (\$98, .9; \$2, .1)$$

Testing Algebraic Models with Error-Filled Data

- We want to test coalescing, restricted branch independence, and transitivity.
- But these properties fail if data contain "error."
- Can violations of these properties be due to "error"? Usually regarded as statistical issue, but requires a THEORY of error.

4 – Error TE model



EU Implies (without error)

	S'	***R'***
S	p_S	0
R	0	$1 - p_S$

Example 1: EU Satisfied?

	S'	***R'***	**sum**
S	65	09	74
R	09	17	26
sum	74	26	100

Example 2: EU Satisfied?

	S'	***R'***	**sum**
S	44	12	56
R	30	14	44
sum	74	26	100

Example 3: EU Satisfied?

	S'	***R'***	**sum**
S	29	06	35
R	36	29	65
sum	65	35	100

TE-1, TE-2, TE-4

- TE-1: All error rates are equal: e = f = e' = f'. (One error rate for all items)
- TE-2: Each item has a different error rate, but e = f and e' = f', even if e ≠ e'.
- TE-4: Four error rates (all free).

4 Error EU TE model (Ex. 3)

- This model fits Hypo Example 3.
- $p_S = 0.5$
- $e = 0.4$, $e' = 0.1$
- $f = 0.1$, $f' = 0.4$
- Thus, Ex. 3 does NOT refute EU, if we allow TE4 errors.

TE models and EU CAN be tested separately

- Yes, we can TEST both TE and EU *if we have replications*.
- Examples 1, 2, or 3 MAY OR MAY NOT agree with EU!
- That is, any of those cases might be consistent with EU. And, any of those cases might violate EU, even those with equal rates of SR’ and RS’.

Examples 1, 2, & 3?

- By separating estimation of ERROR from ASSUMPTION OF EU, we can TEST EU.
- ALL TE models are testable and have EU as a special case, also testable. That is, there are (at least) TWO significance tests: test general TE model and test EU as a special case.

Choice Responses are *not* Independent

- In TE models, choice responses are *not* independent, in general.
- If there is a mixture of true preferences, there will be violations of independence.
- This contrasts with the assumption of iid used by some others. (random utility or “random preference” models assume independence).

Two Choices, Two Reps

- Two Choices, *S* vs. *R* and *S*′ vs. *R*′
- Each choice, can est. errors
- Data: 4 X 4 = 16 cells with 15 df
- Model: 4 "true" probs *SS*' , *SR*' , *RS*' , *RR*'
- Model can have 1, 2, or 4 errors.
- For example, 4 error TE model 15 – 7 df

EU is a Special Case

- The EU model is a special case of TE Models.
- EU Assumption: "true" patterns *SR*' and *RS*' do not occur.
- This model (EU-TE-4) uses 5 parameters: 4 errors and p_S = "true" probability prefer the "Safe" gamble; $p_R = 1 - p_S$

16 cells, 16 equations

$$P(RS', RS') =$$

$$p_{RR'}(1-f)^2(f')^2 +$$

$$p_{RS'}(1-f)^2(1-e')^2 +$$

$$p_{SR'}(e)^2(f')^2 +$$

$$p_{SS'}(e)^2(1-e')^2$$

Allais CC Paradox (*JMP*' 04)

- Choose Between:

  S = (\$40, 0.2; \$2, 0.8)

  R = (\$98, 0.1; \$2, 0.9)

- Choose Between:

  S' = (\$98, 0.8; \$40, 0.2)

  R' = (\$98, 0.9; \$2, 0.1)

- Many *RS*'; i.e., $R > S$ and $S' > R'$

Reanalysis of Birnbaum (2008) *n* = 223 Allais (rev rep)

	RR'	*RS'*	*SR'*	*SS'*
RR'	13	14	5	3
RS'	13	**83**	3	17
SR'	3	0	**1**	3
SS'	4	34	4	23

TE Allais (Rev-rep *n*=223)

Mdl	*f*	*f'*	*e*	*e'*	$p_{SS'}$	$p_{SR'}$	$p_{RS'}$	$p_{RR'}$	*Chi2*
TE-4	0.06	0.13	0.22	0	0.10	0	0.81	0.08	10.92
EU-4	0.50	0.05	0.14	0.50	0.62	(0)	(0)	0.38	50.51
TE-2	= e	= e'	0.20	0.11	0.16	0	0.73	0.16	10.99
EU-2	= e	= e'	0.50	0.30	1.00	(0)	(0)	0	178.3
TE-1	= e	= e	0.16	= e	0.18	0	0.73	0.10	20.60
EU-1	= e	= e'	0.47	= e	1.00	(0)	(0)	0	449.7

RBI Results n = 223 rev rep

	RR'	*RS'*	*SR'*	*SS'*
RR'	30	2	18	9
RS'	7	5	6	8
SR'	17	1	**37**	15
SS'	5	2	14	47

Four Theories Compared

	RBI holds (**cancellation)	RBI fails
Coalescing holds (*combination)	EU, CPT**, OPT*	CPT Inverse-S => *RS'*
Coalescing fails	OPT	TAX, RAM *SR'*

Summary

- TE-4 model can be retained
- TE-2 model good approximation
- TE-1 small but significant violations
- EU model rejected, even with 4 errors
- Two Violations: RBI and Coalescing
- Reject OPT (viol RBI, opposite CPT)
- Reject CPT (viol coalescing)

Available @ my Website

- Birnbaum, M. H., & Quispe-Torreblanca, E. G. (2018). TEMAP2.R: True and error model analysis program in R. *Judgment and Decision Making, 13(5),* 428-440.
- Birnbaum, M. H. (in press?). Bayesian and Frequentist Analysis of True and Error Models. (look in next issue or two of JDM).
- Birnbaum, M. H., & Wan, L. (submitted). MARTER: Markov Chain True and Error Model of Drifting Parameters.

Data from Birnbaum, Schmidt, & Schneider (*JRU*, 2017)

Mdl	f	f'	e	e'	$p_{SS'}$	$p_{SR'}$	$p_{RS'}$	$p_{RR'}$	G^2
TE-4	0	0.10	0.34	0.15	0.58	0.04	0.27	0.11	0.5
EU-4	0	0.50	0.43	0.17	0.71	(0)	(0)	0.29	13.9
TE-2	= e	= e'	0.17	0.14	0.35	0.02	0.49	0.14	2.5
EU-2	= e	= e'	0.50	0.14	0.84	(0)	(0)	0.16	31.1
TE-1	= e	= e'	0.17	0.15	0.35	0.02	0.49	0.14	2.6
EU-1	= e	= e'	0.17	0.15	0.70	(0)	(0)	0.30	112.3

Website URL

- http://psych.fullerton.edu/mbirnbaum/birnbaum.htm
- Email: mbirnbaum@fullerton.edu

Key: Use reps to estimate errors

- The key to this approach: use replicates to estimate (constrain) error rates.
- Prev. use of errors as parameters to make EU fit meant that model deviations were being attributed to error.
- Even the four-error model becomes testable in this approach.

Coalescing and models

- EU implies coalescing
- RDU, RSDU, CPT imply it
- Original PT “combination”
- SWU, SWAU, strip’d PT violate
- TAX & RAM violate coalescing

BSS-Results (refute EU with TE-4)

	SS'	*SR'*	*RS'*	*RR'*
SS'	36	7	21	3
SR'	5	4	1	3
RS'	18	4	69	11
RR'	1	2	12	14

BSS-Results (split form, RBI)

	SS'	*SR'*	*RS'*	*RR'*
SS'	42	24	4	1
SR'	17	21	2	16
RS'	7	4	14	7
RR'	1	6	6	39